# Cheat Sheet of Mathemtical Notation and Terminology

## Logic and Sets

| Notation | Terminology | Explanation and Examples |
|---|---|---|
| $a := b$ | *defined by* | The object $a$ on the side of the colon is defined by $b$. |
| | | *Examples:* $x := 5$ means that $x$ is defined to be 5, or $f(x) := x^2 - 1$ means that the function $f$ is defined to be $x^2 - 1$, or $A := \{1, 5, 7\}$ means that the set $A$ is defined to be $\{1, 5.7\}$. |
| $S_1 \Rightarrow S_2$ | *implies* | Logical implication: If statement $S_1$ is true, then statement $S_2$ must be true. We say $S_1$ *is a sufficient condition for* $S_2$ or $S_2$ *is a necessary condition for* $S_1$. |
| | | *Examples:* $(n \in \mathbb{N}$ even$) \Rightarrow (n^2$ even$)$. |
| $S_1 \Leftrightarrow S_2$ | *equivalent to* | Logical equivalence: If statement $S_1$ is true, then statement $S_2$ must be true, and vice versa. We say $S_2$ *is a necessary and sufficient condition for* $S_1$. |
| | | *Examples:* $(\ln x > 0) \Leftrightarrow (x > 1)$. |
| $\exists$ | *there exists* | Abbreviation for *there exists* |
| $\forall$ | *for all* | Abbreviation for *for all* |
| $\{\dots\}$ | *set* | The "objects" listed between the curly brackets are members of the set being defined. |
| | | *Examples:* $\{0, 2, 5, 7\}$, $\{2 + i, 7 - \sqrt{5}\}$,  The elements of a set can be any kind of objects such as numbers, functions, points, geometric objects or other. |
| $a \in A$ | *element of* | $a$ is an element of the set $A$, that is, $a$ is in the set $A$. |
| | | *Examples:* $\pi \in \mathbb{R}$, $4 \in \{1, 4, 7\}$,  |
| $\emptyset$ or $\{\}$ | *empty set* | The special set that does not contain any element. |
| $\{x \mid \text{property}\}$ | *set of … with …* | Notation indicating a set of elements $x$ satisfying a certain property. |
| | | *Examples:* $\{n \in \mathbb{N} \mid n \text{ is even}\}$, where $n \in \mathbb{N}$ is the typical element and the property satisfied is that $n$ is even. $\{x^2 \mid x \in \mathbb{N}\}$, where the typical member is a square of some number in $\mathbb{N}$. |
| $A \subseteq B$ | *subset of* | The set $A$ is a subset of $B$, that is, every element of $A$ is also an element of $B$. More formally: $b \in B \Rightarrow b \in A$. |
| | | *Examples:* $\mathbb{Q} \subseteq \mathbb{R}$, $\{1, 4, 7\} \subseteq \{1, 2, 3, 4, 5, 6, 7\}$ |
| $A \cup B$ | *union* | The set of elements either in $A$ or in $B$. More formally: $(x \in A \cup B) \Leftrightarrow (x \in A$ or $x \in B)$. |
| | | *Examples:* $\{1, 4, 7\} \cup \{4, 5, 8\} = \{1, 4, 5, 7, 8\}$ (elements are not repeated in a union if they appear in both sets!) |
| | | *Note:* We can look at a union of an arbitrary collection of sets: The set of objects that appear in at least one of the sets in the collection. |
| $A \cap B$ | *intersection* | The set of elements that are in $A$ and in $B$. More formally: $(x \in A \cap B) \Leftrightarrow (x \in A$ and $x \in B)$. |
| | | *Examples:* $\{1, 4, 7\} \cap \{1, 2, 3, 5, 6, 7\} = \{1, 7\}$ |
| | | *Note:* We can look at the intersection of an arbitrary collection of sets: The set of objects that appear in every set in the collection. |
| $A \setminus B$ | *complement* | The set of elements that are in $A$ but not in $B$. More formally: $(x \in A \setminus B) \Leftrightarrow (x \in A$ and $x \notin B)$. |
| | | *Examples:* $\{1, 4, 5, 7\} \setminus \{1, 2, 3, 6, 7\} = \{4, 5\}$ |

## Interval notation

| Notation | Terminology | Explanation and Examples |
|---|---|---|
| $[a, b]$ | *closed interval* | If $a, b \in \mathbb{R}$ with $a \leq b$ the closed interval is the set $\{x \in \mathbb{R} \mid a \leq x \leq b\}$ |
| | | *Examples:* $[-3, 5]$ is the set of real numbers between $-3$ and 5, including the endpoints $-3$ and 5. |
| $(a, b)$ | *open interval* | If $a, b \in \mathbb{R}$ with $a \leq b$ the open interval is the set $\{x \in \mathbb{R} \mid a < x < b\}$ |
| | | *Examples:* $(-3, 5)$ id the set of real numbers between $-3$ and 5, excluding the endpoints $-3$ and 5. |
| $[a, b)$ or $(a, b]$ | *half open interval* | If $a, b \in \mathbb{R}$ with $a \leq b$, $[a, b)$ is the set of all numbers between $a$ and $b$ with $a$ included and $b$ excluded. In case of $(a, b]$ the endpoint $a$ is excluded and $b$ is included. |
| | | *Examples:* $[-3, 5)$ is the set of real numbers between $-3$ and 5, including $-3$ but excluding 5. For $(-3, 5]$ the endpoint $-3$ is excluded and 5 is included. |
| $[a, \infty)$ or $(-\infty, a]$ | *closed half line* | If $a \in \mathbb{R}$, then $[a, \infty)$ is the set of real numbers larger than or equal to $a$, and $(-\infty, a]$ is the set of real numbers less than or equal to $a$ |
| $(a, \infty)$ or $(-\infty, a)$ | *open half line* | If $a \in \mathbb{R}$, then $(a, \infty)$ is the set of real numbers strictly larger than $a$, and $(-\infty, a)$ is the set of real numbers strictly less than $a$ |
| | | *Examples:* $(0, \infty)$ set of all positive real numbers; $(-\infty, 5]$ set of all real numbers less than or equal to 5. |

## Functions

| Notation | Terminology | Explanation and Examples |
|---|---|---|
| $f : A \to B$ | *function* | A function $f$ from the set $A$ to the set $B$ is a rule that assigns every element $x \in A$ a unique element $f(x) \in B$. The set $A$ is called the *domain* and represents all possible (or desirable) "inputs", the set $B$ is called the *codomain* and contains all potential "outputs". |
| $x \mapsto f(x)$ | *is mapped to* | The function maps $x$ to the value $f(x)$. |
| | | *Examples:* $g : \mathbb{R} \to \mathbb{C}, \theta \mapsto g(\theta) := e^{i\theta}$. A function from $\mathbb{R}$ to $\mathbb{C}$ given by $e^{i\theta}$; $f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) := 1 + x^2$. A function from $\mathbb{R}$ to $\mathbb{R}$ given by $1 + x^2$; $h : \mathbb{C} \to [0, \infty), z \mapsto h(z) := \|z\|$. A function from $\mathbb{C}$ to $[0, \infty)$ given by $\|z\|$. |
| $\text{im}(f)$ | *image or range* | The set of values $f : A \to B$ attains: $\text{im}(f) := \{f(x) : x \in A\} \subseteq B$. |
| | | *Examples:* $f : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) := x^2$. The codomain is $\mathbb{R}$, the image or range is $[0, \infty)$. |
| | *surjective or onto* | A function $f : A \to B$ is called *surjective* if $\text{im}(f) = B$, that is, the codomain coincides with the range. More formally: For every $b \in B$ there exists $a \in A$ such that $f(a) = b$. |
| | | *Note:* $f : A \to \text{im}(f)$ is always surjective. The choice of codomain is quite arbitrary. We often just state the general objects rather than the image or range. For instance function values are in $\mathbb{R}$ if we are not intested in the image. |
| | *injective or one-to-one* | A function $f : A \to B$ is called *injective* if $\text{im}(f) = B$, that is, every point in the image comes from exactly one point in the domain $A$. More formally: If $a_1, a_2 \in A$ are such that $f(a_1) = f(a_2)$, then $a_1 = a_2$. |
| | *bijective* | A function $f : A \to B$ is called *bijective* if it is surjective and injective. |
| $f^{-1}$ | *inverse function* | A function $f : A \to B$ is called *invertible* if it is bijective. The inverse function $f^{-1} : B \to A$ is defined as follows: Given $b \in B$ take the unique point $a \in A$ such that $f(a) = b$ and set $f^{-1}(b) := a$ (by surjectivity such $a$ exists, by injectivity it is unique). |